About the possibility of five-dimensional effective theories for low-energy QCD

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Abstract

The AdS/QCD models suggest an interesting idea that the effective theory of low-energy QCD may be formulated as a 5-dimensional field theory in the weak coupling regime in which the fifth coordinate plays a role of inverse energy scale. Taking the point of view that this is just an efficient parametrization of the non-perturbative dynamics of strong interactions, we discuss on a qualitative level an alternative possibility for a simpler 5-dimensional parametrization of main phenomena in the low-energy QCD. We propose to interpret the effect of chiral symmetry breaking as an effective appearance of compactified extra dimension with the radius of the order of inverse scale of chiral symmetry breaking. Following some heuristic arguments two dual scenarios for the emergence of the excited light mesons are introduced: In the first scenario, the meson resonances are interpreted as the effects of Kaluza-Klein excitations of quarks inside mesons, in the second one, as the formation of gluon strings wound around the compactified dimension an appropriate number of times. Matching of these scenarios permits to express the slope of radial Regge trajectories through the order parameters of the chiral symmetry breaking, with the compactification radius being excluded. This example shows qualitatively that the extra dimension may play an auxiliary role providing a short way for deriving new relations.

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1 Introduction

Recently, the attempts to develop a systematic non-perturbative approach to strongly coupled gauge theories have led to necessity to invoke an extra spatial dimension — it is required by the AdS/CFT correspondence [1]. In application to QCD, the efforts to find an appropriate five-dimensional dual theory formulated on an anti-de-Sitter (AdS) gravity background are referred to as AdS/QCD [2–4].

A genuine 5d holographic dual for QCD hardly can exist (see, *e.g.*, the recent discussions in [5]), nevertheless, the AdS/QCD approach turned out to be surprisingly successful in description of low-energy observables. Thus, an important lesson from studies of AdS/QCD models consists in the fact that the low-energy dynamics of strong interactions may be parametrized geometrically if one introduces one extra spatial dimension.

One can assume that this fact has nothing to do with the original AdS/CFT correspondence but reflects something that we do not yet understand. It is therefore interesting to find alternative extra dimensional parametrizations of non-perturbative dynamics of QCD which could in a simpler way compared to AdS/QCD describe the hadron spectrum and related physics.

The identification of the radial excitations of light mesons with the tower of Kaluza-Klein modes is at the heart of AdS/QCD models. This observation can be used to simplify significantly the 5-dimensional description of meson excitations and to make it less model dependent — instead of warped extra dimension we will consider a compactified extra dimension of a radius R, the 5d space-time in this picture is therefore flat with the topology $M^4 \times S^1$. In addition, we will assume that interactions in such a theory are weak to the extent that they can be neglected when calculating the mass spectrum in the first approximation. From the 4d point of view, a free 5d particle in the space $M^4 \times S^1$ looks as an infinite tower of Kaluza-Klein modes with the masses $m_n \sim n/R$ (see, *e.g.*, a review [6]), *i.e.* we recover immediately the asymptotic spectrum of the hard-wall AdS/QCD models [2]. In reality, however, one expects the Regge spectrum

$$m^2 \sim n. \tag{1}$$

In order to obtain the relation (1) within the AdS/QCD models one needs to assume a rather nontrivial 5d background [3] (see, however, [7]). The crucial step in our approach is to replace this assumption by another one that could be justified within the real QCD. We assume that *all* masses square of light mesons are linear in the current quark mass,

$$m^2 \sim C_1 + C_2 m_q, \tag{2}$$

where C_1 and C_2 are some constants. We will show with the help of simple model-independent estimates that the radius R is of the order of $\Lambda_{\text{CSB}}^{-1}$, where Λ_{CSB} is the scale of the Chiral Symmetry Breaking (CSB), about 1 GeV, and is related to the order parameters of CSB. This leads to the interpretation of the CSB as an effective emergence of one compactified extra spatial dimension.

The paper is organized as follows. In Sect. 2, we provide arguments concerning the validity of relation (2) and discuss the spectrum of excited states. Then we propose, in Sect. 3 and Sect. 4, two complementary scenarios for the emergence of linear spectrum, $m_n^2 \sim n$: In the first case, the excitations are interpreted as the Kaluza-Klein modes of quarks, in the second one, the n-th excitation represents a gluon string between the quark and antiquark that is wound n times around a compactified extra dimension. Alternative interpretations for the compactification radius R are discussed in Sect. 5. In Sect. 6, we compare the proposed scenarios with the AdS/QCD approach and other extra dimensional models. We conclude in Sect. 7.

2 Meson mass vs. quark mass

The origin of light hadrons is believed traditionally to be hidden in the highly complicated non-perturbative strong interactions described by QCD. The only well-established relation between the parameters of the QCD Lagrangian and the hadron masses is the Gell-Mann–Oakes–Renner (GOR) one [8], in the limit of exact $SU(2)$ isospin symmetry it takes the form,

$$m_\pi^2 = -\frac{2m_q \langle \bar{q}q \rangle}{f_\pi^2}, \tag{3}$$

where m_q is the current quark mass, $f_\pi = 92.4$ MeV [9] represents the weak pion decay constant, and $\langle \bar{q}q \rangle$ is the quark condensate, the latter two quantities are phenomenological parameters. Relation (3) comes from the interpretation of pion as the pseudogoldstone boson of spontaneously broken chiral invariance of QCD in the limit $m_q = 0$. Since the term $m_q \bar{q}q$ enters the QCD Lagrangian, the form of relation (3) looks quite natural. In addition, the linear dependence $m_\pi^2(m_q)$ can be obtained heuristically — since the relativistic field equations include the linear fermion masses and the square boson ones, the simplest nontrivial relation $m_\pi^2(m_q)$ is just a linear function. However, m_q does not represent a renorminvariant quantity while m_π^2 does (it is an observable), but one can build a renorminvariant quantity $m_q \langle \bar{q}q \rangle$ if the CSB takes place (*i.e.* if $\langle \bar{q}q \rangle \neq 0$), so $m_\pi^2 \sim m_q \langle \bar{q}q \rangle$. The relation $m_\pi \sim f_\pi$ cannot take place as $f_\pi \neq 0$ in the limit $m_\pi = 0$, $m_q = 0$. Thus, one arrives at

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relation (3) just by dimensional analysis plus the requirement of renormin-variance (factor 2 appears from the sum $m_u + m_d$, the result can depend only on this sum in the linear approximation by symmetry reasons). The linear dependence $m_\pi^2 = \Lambda m_q$ was checked on lattice [10] and up to all available m_q in the lattice simulations the agreement was quite prominent.

As to the spectrum of other light nonstrange mesons, the matter did not go here beyond different models. In the first approximation, the spectrum of masses square is equidistant,

$$m_n^2 = an + m_0^2, \qquad n = 0, 1, 2, \ldots, \tag{4}$$

where n denotes either spin (the Regge trajectories) or "radial" quantum number (*i.e.* it enumerates the daughter Regge trajectories), the slope a is nearly the same for both cases while the intercept m_0^2 is channel-dependent. This picture agrees very well with the phenomenology [11] (see also discussions in [12–16]). The observations of behaviour (4) gave rise to the idea of strings and up to now the conjecture that QCD in the hadronization regime is dual to some string theory is still alive. In this case, the slope a is related to the string tension σ as

$$a = 2\pi\sigma. \tag{5}$$

We note that the intercept m_0^2 should contain a contribution arising from the term $m_q \langle \bar{q}q \rangle$ since the arguments for its existence are quite general. This is our first important observation.

We consider the spectrum of mesons as various excitations of the pion. Since the (charged) pion lives near 10^{16} times as long as its excitations, these excitations can be regarded as sudden perturbations of the lightest bound state in QCD caused, say, by a collision with a hadron. This hierarchy of time scales is our second important observation: As is known from the Quantum Mechanics the wave function of the bound state is not changed for such a short period of time, hence, the relations involving the bound state do not change.

3   Model 1: Kaluza-Klein scenario

Let us introduce one extra spatial dimension with the topology S^1 and radius R. According to the Kaluza-Klein theories (for a review see, *e.g.*, [6]) a particle of mass m excited along the 5-th compactified dimension looks like a usual 4-dimensional particle with the mass

$$m_n^2 = m^2 + \frac{n^2}{R^2}, \qquad n = 0, \pm 1, \pm 2, \ldots \tag{6}$$

Assume now that one of quarks forming the lightest meson — the pion — can be excited along the 5-th dimension and the corresponding Kaluza-Klein modes give rise to the observed spectrum of light nonstrange mesons (since we are dealing semiclassically with a two-body system the excitation of both quarks can be always represented as the excitation of one quark only). Due to the linear dependence on quark mass, $m_\pi^2 = \Lambda m_q$ and the hierarchy of time scales discussed in the previous Section, relations (3) and (6) should then dictate the following spectrum of excitations,

$$m_n^2 = -\frac{\langle \bar{q}q \rangle}{f_\pi^2} \left(m_q + \sqrt{m_q^2 + \frac{n^2}{R^2}} \right) + \tilde{m}_0^2, \qquad (7)$$

where $\tilde{m}_0^2$ is a channel-dependent constant parametrizing various side effects which could exert influence on the determination of resonance position. In the light flavor sector one can neglect the contribution of current quark mass m_q if $n \neq 0$, the spectrum is then linear. Comparing the slopes in Eqs. (4) and (7) we arrive at the following matching condition,

$$R = -\frac{\langle \bar{q}q \rangle}{f_\pi^2 a}. \qquad (8)$$

Thus, the size of extra dimension and parameters of non-perturbative QCD turn out to be remarkably related. For typical phenomenological values of the quark condensate[2] and the slope, $\langle \bar{q}q \rangle = -(0.235 \text{ GeV})^3$ and $a = (1.1 \text{ GeV})^2$, relation (8) yields the estimate $R \approx 1.3 \text{ GeV}^{-1}$ (or 0.25 fm). This estimate and the fact that R is related to the order parameters of CSB suggest that the compactification radius can be related to Λ_{CSB}, $R \simeq \Lambda_{\text{CSB}}^{-1}$.

The validity of the idea above is supported by the following estimate. The lifetime (inverse full decay width) of excited mesons is just the time of "living" in the extra dimension. The typical decay widths of hadron resonances show that their lifetime is of the order of $\tau \sim 10^{-24}$ s. Since the quarks inside the light mesons are ultrarelativistic they propagate with the speed of the order of $c = 3 \cdot 10^8$ m/s. Hence, the size of extra dimension can be estimated as $c\tau \sim 0.3$ fm that is in accord with our previous estimate.

4   Model 2: Gluon string scenario

Perhaps the most natural model of light mesons reproducing the linear spectrum (4) is the model of effective gluon string — a flux tube of chromoelec-

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[2]The quantities $\langle \bar{q}q \rangle$ and R are not renorminvariant, according to Eq. (7) renorminvariant is the combination $\langle \bar{q}q \rangle R^{-1}$. We make estimates at the scale about 1 GeV and neglect the running of those quantities with energy in the resonance region in what follows.

tric field of constant density σ — stretched between the quark an antiquark. Within such models, spectrum (4) is obtained as a result of semiclassical quantization of the string (see, *e.g.*, [12] and references therein). A drawback of these models is that the size of the mesons grows linearly with the mass, making the linear size of the highly excited states unrealistically large while in reality this size is approximately the same as the size of pions. To overcome the difficulty one may assume that the gluon string is stretched not in the observable 3 spatial dimensions but in the extra dimension. If the string is n times wound around the compactified extra dimension, the quark-antiquark system acquires the mass[3] $M = \pi R n \sigma = a R n / 2$ (see Eq. (5)) which can be interpreted as an effective excitation energy of one of quarks. Applying the arguments of the previous Section we obtain that when the CSB occurs the spectrum should be

$$m_n^2 = -\frac{\langle \bar{q} q \rangle}{f_\pi^2} \left(2 m_q + \frac{a R n}{2} \right) + \tilde{m}_0^2. \tag{9}$$

Matching of Eq. (9) to Eq. (4) yields

$$R^{-1} \simeq -\frac{\langle \bar{q} q \rangle}{2 f_\pi^2} \equiv \frac{B}{2}, \tag{10}$$

where $B = -\frac{\langle \bar{q} q \rangle}{f_\pi^2} \approx 1.5$ GeV represents an important phenomenological parameter in the low-energy effective theories of QCD.

Relation (10) leads to the estimate $R \approx 1.3$ GeV^{-1}. Thus, the radius of compactified dimension in Model 2 is practically equal to that of Model 1. This coincidence seems to be not fortuitous — it strongly suggests that both models are complementary, hence, they can be matched to each other. Comparing Eq. (9) with Eq. (7) and neglecting the current quark mass m_q we arrive at the relation

$$a \simeq 2 R^{-2}. \tag{11}$$

Substituting Eq. (10) into Eq. (11) we obtain

$$\sqrt{a} \simeq -\frac{\langle \bar{q} q \rangle}{\sqrt{2} f_\pi^2}. \tag{12}$$

Relation (12) shows explicitly how the spectrum of highly excited states is determined by the order parameters of the chiral symmetry breaking in QCD.

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[3]In order to avoid the chirality problem we must replace the circle formed by the 5-th dimension by the orbifold — a circle with identified opposite points [6], in this way the physical interval extends a length πR.

It must be emphasized that the radius R of compactified dimension disappears in the final expression (12). This example demonstrates qualitatively how the extra dimensions can be exploited as an auxiliary tool for finding new relations between parameters of low-energy QCD.

5   Alternative interpretations for compactification radius

The interpretation of R^{-1} as Λ_{CSB} is suggestive but not unique. Since according to our estimates $R^{-1} \approx 0.76$ GeV, it might be possible to identify R^{-1} with the mass of ρ-meson, $m_\rho \approx 0.78$ GeV [9].

Another tempting albeit highly speculative interpretation of R in Model 1 consists in identifying R^{-1} with the constituent mass M_{con}. This interpretation could be deduced as follows.

Instead of Eq. (4) we should write

$$m_n^2 = m_\rho^2 n, \qquad n = 0, 1, 2, \ldots, \tag{13}$$

where the states alternate in parity. This is nothing but the spectrum of the Ademollo–Veneziano–Weinberg dual amplitude [17] and is resembling the asymptotics of dim2 QCD spectrum in the large-N_c limit [18]. Simultaneously, one achieves agreement in the slope value with the heavy-light quarkonia [19] as long as $m_\rho^2 \simeq a/2$ numerically and in many models (see, e.g., [20]). Within Model 1, the radius R in Eq. (8) is then enhanced by a factor of 2. We observe then that the mass of the first Kaluza-Klein mode of light quark (let be $m_q^{(1)}$) becomes close to the constituent quark mass $M_{\text{con}} \simeq 0.3$ GeV (say, for $\langle \bar{q}q \rangle = -(0.25 \text{ GeV})^3$ one has $R \approx 3$ GeV^{-1} that gives $m_q^{(1)} \approx 0.33$ GeV). The identification of $m_q^{(1)}$ with M_{con} provides a rather unexpected prospects for the explanation of still enigmatic success of nonrelativistic constituent quark model. In addition, it suggests that since the ρ-meson is the first non-goldstone excitation the formula for the ρ-meson mass is just the GOR relation (3) in which one of quark masses m_q is replaced by M_{con},

$$m_\rho^2 \simeq -\frac{(m_q + M_{\text{con}})\langle \bar{q}q \rangle}{f_\pi^2}. \tag{14}$$

This relation is fulfilled with about 5-10% accuracy, in this regard it is interesting whether it could be derived from some dynamical model. Relation (14) is another example of new relations which might by found with the help of extra dimensions.

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6 Discussions

Let us point out some important distinctions between the presented scheme that we will refer to as Kaluza-Klein/QCD (KK/QCD) approach (Model 1) and the AdS/QCD one. First of all, the AdS/QCD ideology is motivated by the AdS/CFT correspondence [1] while we do not see such an inspiring analogue for the KK/QCD approach. However, QCD is neither conformal no supersymmetric and for this reason presently the AdS/QCD has a status of speculative hypothesis that for some unknown reasons works surprisingly well in description of hadron physics up to 2 GeV. In this regard, the level of speculations in AdS/QCD and in KK/QCD is close. Second, the AdS/QCD approach relates the QCD dynamics with metrics on the boundary of AdS space that is induced by the gravity in the bulk. In a sense, the KK/QCD approach is also related with a higher dimensional gravity. The matter is that the KK-reduction of gravity to four dimensions leads to appearance of radion (modulus field) whose v.e.v. fixes the radius of extra dimension [6]. As within KK/QCD this radius is fixed by relation (8) we arrive at intriguing prospects to relate the non-perturbative dynamics of strong QCD to various mechanisms of modulus stabilization proposed in the literature (see, *e.g.*, [6]). In a sense, the relation between AdS/QCD and KK/QCD resembles the relation between the Rundall-Sundrum models [21] and the ADD models [22] of extra dimensions. In both cases new high precision experiments in spectroscopy of light hadrons below 2.5 GeV are needed to learn about the underlying geometry.

Since the curvature of 5d space in KK/QCD is zero (or least is negligible), there is an inspiring possibility that the extra dimension represents a physical reality. If this is the case, the KK/QCD scenario differs substantially from the ADD one. First, the size of extra dimension is not sub-millimetric (up to 0.1 mm according to the ADD-phenomenology), it is near 1 fm. This spoils the main model-independent feature of the ADD-models — the prediction of high-multiplicity emission of light Kaluza-Klein gravitons. Second, the excited quarks do not escape into extra dimensions because their wavelength $\lambda_n \sim 1/m_n \sim R$ and the size of the compact dimension are comparable, *i.e.* they represent genuine Kaluza-Klein excitations with momentum in compact dimension, such states are interpreted from the 4-dimensional point of view as particles of mass $\sim 1/R$ that are still localized in our 4 dimensions. This solves a possible problem with the conservation of electric charge in our 4-dimensional world.

We have restricted ourselves by the mesons composed of light nonstrange quarks. Concerning the other hadrons, the discussion will be practically unchanged if we include light mesons containing the strange quark since the

GOR relation (3) remains valid and the corresponding Regge trajectories are also linear with practically equal slope. As to the light baryons, their spectrum can be well fitted by a formula like (4) with the same slope [23]. This gives a certain hope that the dual Kaluza-Klein mechanism is also relevant here, but the situation may be more tricky since we do not have any GOR-like relation for the ground state mass. In the heavy-light quarkonia, one could expect similar Kaluza-Klein excitations of the light constituent. However, the slope of the corresponding trajectories seems to be half of the value for the light-light quarkonia [19] (see, however, the discussions of Eq. (13)). Concerning the rest of the hadrons, we have nothing to say presently.

7   Conclusions

On a rather heuristic level, we have discussed possible geometrical interpretations of such non-perturbative aspects of QCD as the chiral symmetry breaking and the meson spectrum. It is argued that the effective theories for strong QCD could be searched on the base of 5d field theories with one compactified extra dimension. In the region of low and intermediate energies, such models represent an alternative to the usual AdS/QCD bottom-up approach where the extra dimension is warped. Within the conjectured scenario, the abundance of hadron resonances composed of light quarks is interpreted as the effect of the Kaluza-Klein excitations of the quarks inside mesons or, alternatively, as the formation of gluon strings wound around compactified extra dimension. The ensuing models entail some simple relations between the size of extra dimensions and phenomenological quantities characterizing the non-perturbative QCD. The matching of this picture to the phenomenology is achieved for a typical size of extra dimension of the order of 1 fm. If the 5th dimension represents a physical reality the outlined picture suggests that its size and possibly another characteristics could be probed by high precision experiments in hadron spectroscopy below 2.5 GeV.

Unfortunately, the presented arguments are not yet backed up by a careful theoretical analysis of a specific field theory. It would be extremely interesting to construct an explicit example of such a theory.

Acknowledgments

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